"Fishbones, Wheels, Eyes, and Butterflies: Is There a Unified Account of Mathematical and Physical Modeling?"

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Abstract

David Hilbert saw the connection between physics and mathematics as the "nerve" that invigorates mathematical reasoning, and argued that cutting it off would stifle the growth of mathematical research. Quine and Putnam argued that mathematics is indispensable to physical reasoning. Recently, Chris Pincock has argued for mathematical realism based on the use of mathematical models of natural processes and phenomena. I would argue that such accounts presuppose that mathematics can be employed, or applied, in physical reasoning. This poses a paradox: when mathematics is applied in physics, are the mathematical quantities or magnitudes involved still a set of independent, formal relationships or structures? Or does applied mathematics become physics? If it does, how do we remove the mathematical relationships from the physical theories at issue again, to carry out Hilbert's mission of enriching pure mathematics? How do we evaluate pure or abstract mathematical magnitudes that seem to take on a physical significance, when applied, that is not easy to explain?

Conformal maps are transformations that preserve local angles. As such, their study can be viewed as part of the study of purely geometrical - better, topographical - functions. But conformal maps are well known to bridge the divide between pure and applied mathematics. Recently, conformal mapping has been used to provide exact solutions to the Navier-Stokes equations for fluid flows (Bazant and Moffatt 2005). Bazant and Moffatt provide a number of exact solutions that depend straightforwardly on the Reynolds number, a dimensionless constant that is the ratio of inertial to viscous forces within a fluid. They conclude, "These solutions provide mathematical insights into the Navier–Stokes equations and physical insights into ways that vorticity may be confined. They also provide stringent tests for the accuracy of numerical simulations" (p. 63). Nonetheless, the solutions, and the models that result, depend on a dimensionless constant, and thus the question arises: how does a constant with no obvious physical meaning constrain or even allow for the application of mathematical models to physical systems? How do we construct meaningful, testable physical models using abstract quantities and transformations - conformal maps, dimensionless constants -

that are, in some sense, non-natural? As Quine and Putnam observe, such quantities may even be indispensable to providing a model of such systems, which makes the success of the models even more mysterious.

The paper concludes with an analysis of the dual role of geometrical and topographical reasoning in applied mathematics: providing understanding or explanation, on the one hand, and theory testing and explorative reasoning and experiment, on the other. In mathematics, just as in physics, the success or failure of techniques and experiments in proof can suggest ways to alter or to strengthen the structure of existing theory. In principle, the method of testing and revision could be developed as a unified, general heuristic for mathematical and physical modeling. The question of how to descend to more specific recommendations for the modeler is complex, however. It is an open question, at this point, whether there is any specific set of constraints or methods common to mathematical and to physical models. If that is the case, though, then the question remains: how do physical models rigged with abstract, non-natural components work in practice?